(Preprint) AAS 16-267

IMPACT RISK ANALYSIS OF NEAR-EARTH ASTEROIDS WITH MULTIPLE SUCCESSIVE EARTH ENCOUNTERS

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Accurate estimation of the impact risk associated with hazardous asteroids is essential for planetary defense. Based on observations and the risk assessment analyses of those near-Earth objects (NEOs), mission plans can be constructed to deflect/disrupt the body if the risk of an Earth impact is large enough. Asteroids in Earth resonant orbits are particularly troublesome because of the continuous threat they pose to the Earth in the future. The problem of analyzing the impact risk associated with NEOs on a close-encounter with the Earth has been studied in various formats over the years. However, the problem of multiple, successive encounters with the Earth needs to be further investigated for planetary defense. Incorporating methods such as analytic encounter geometry, target B-planes, analytic keyhole theory, and numerical simulations presents a new computational approach to accurately estimate the impact probability of NEOs, especially those in Earth resonant orbits.

INTRODUCTION

Asteroids impacting the Earth is a real and ever-present possibility. The ability to determine the likelihood of an impact is a necessity in order to have the chance to counteract their imposing threat. When it comes to near Earth asteroids, there are three major components to any design or mission involving these asteroids: identification, orbit determination, and mitigation. A lot of effort has been made to identify the threats to the Earth as early as possible. At the Asteroid Deflection Research Center, there has been a lot of work done on the mitigation component by studying potential mission designs to deflect and/or disrupt hazardous asteroids.[1–3] The focus of this paper is on the second of the three components - the orbit determination of the asteroids with multiple, successive Earth encounters.

The ability to determine the orbit of a potentially hazardous near-Earth asteroids is very important. In previous papers,[1–3] techniques including a high precision gravitational simulator, target planes, and keyholes have been used to help evaluate the orbits that an asteroid has the potential of getting into and their associated impact probability. Upon acquisition of an asteroid, a high-fidelity gravitational model can be used to propagate an asteroid's state into the future to see if/when it would come in close proximity to another body. Encounters with planets would most likely result in the asteroid's orbit changing. Based on the encounter's geometry, the resulting heliocentric orbit could be in resonance with the encounter planet that would result in another encounter or impact. Using the target B-plane and keyhole theory, an estimate on the current and future impact probability

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between the asteroid and the Earth can be made. In this paper, the previously established methods will be applied to asteroid 2012 TC4 - a near-Earth asteroid that has two successive encounters with the Earth in October 2012 and October 2017.

ORBIT DETERMINATION

By decree of Congress in 1990, NASA had to find ways to increase the rate of discovery of near-Earth objects (NEOs). Through such efforts, objects of significant size have been found on occasion to be on a potential Earth-impacting trajectory. Often requiring high-fidelity propagation models, containing the effects of non-gravitational orbital perturbations such as solar radiation pressure (SRP), the accurate prediction of such Earth-impacting trajectories could be found. Such highly precise asteroid orbits allows mission designers to take advantage of more specific mission planning, higher certainty of the target's location, and more accurate impact probability.

Orbit Simulation

The orbital motion of an asteroid is governed by the so-called Standard Dynamical Model (SDM). The Newtonian n-body equations of motion (EOMs) used for the orbital propagation of asteroids takes the form[4]

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r} + \sum_{k=1}^{n} \mu_k \left(\frac{\vec{r}_k - \vec{r}}{|\vec{r}_k - \vec{r}|^3} - \frac{\vec{r}_k}{r_k^3} \right) + \vec{f} \tag{1}$$

where $\mu = GM$ is the gravitational parameter of the Sun, n is the number of perturbing bodies, μ_k and $\vec{r}_k$ are the gravitational parameter and heliocentric position vector of perturbing body k, respectively, and $\vec{f}$ represents other non-conservative orbital perturbation acceleration. The three most well-known are solar radiation pressure (SRP), relativistic effects, and the Yarkovsky effect, the former two being the most prevalent effects. Solar radiation pressure provides a radial outward force on the asteroid body from the interaction of the Sun's photons impacting the asteroid surface. The SRP model is given by

$$\vec{a}_{SRP} = (K)(C_R)\left(\frac{A_R}{m}\right)\left(\frac{L_S}{4\pi c r^3}\right)\vec{r} \tag{2}$$

where $\vec{a}_{SRP}$ is the solar radiation pressure acceleration vector, C_R is the coefficient for solar radiation, A_R is the cross-sectional area presented to the Sun, m is the mass of the asteroid, K is the fraction of the solar disk visible at the asteroid's location, L_S is the luminosity of the Sun, c is the speed of light, and $\vec{r}$ and r is the distance vector and magnitude of the asteroid from the Sun, respectively. The relativistic effects of the body are included because for many objects, especially those with small semi-major axes and large eccentricities, those effects introduce a non-negligible radial acceleration toward the Sun. One form of the relativistic effects is represented by

$$\vec{a}_R = \frac{k^2}{c^2 r^3}\left[\frac{4k^2\vec{r}}{r} - \left(\dot{\vec{r}} \cdot \dot{\vec{r}}\right)\vec{r} + 4\left(\vec{r} \cdot \dot{\vec{r}}\right)\dot{\vec{r}}\right] \tag{3}$$

where $\vec{a}_R$ is the acceleration vector due to relativistic effects, k is the Gaussian constant, c is the speed of light, $\vec{r}$ is the position vector of the asteroid, and $\dot{\vec{r}}$ is the velocity vector of the asteroid.[6] In the case of near-Earth asteroids, the acceleration term due to relativistic effects is not necessary, but is included herein for completeness.

In the case when the Earth is considered the central body, as is done when dealing with the flyby of the asteroid, another perturbation must be added to the model in order to maintain accuracy. The extra perturbation would be due to the Earth's oblateness, known as the J2 gravity perturbation. This perturbation has to be taken into account for orbits about Earth because of the non-uniformity of Earth's surface, which is assumed when dealing with the equations of motion. For most circumstances, the equations of motion are fine as given above in Eq. (1) because the distances between the simulated body and any other body is large enough that they can be assumed to be a point mass. During planetary flybys, particularly flybys with the Earth, the simulated body can pass close enough to the planet that it can't be assumed to be a uniform sphere that can mathematically be represented as a point mass. When considering orbits about Earth, an additional potential energy term must be added to the overall potential energy of the planet, as

$$V(x, y, z) = \frac{\mu_\oplus}{r} + \frac{J_2 \mu R_\oplus^2}{2r^3}\left[3\left(\frac{z}{r}\right)^2 - 1\right] \tag{4}$$

where $r = \sqrt{x^2 + y^2 + z^2}$, with $\mu_\oplus$ being the Earth's gravitational parameter, $J_2 = 1.082617 \times 10^{-3}$ is the second zonal harmonic coefficient, and $R_\oplus$ is the mean equatorial radius of the Earth. To describe the motion of a body with respect to the Earth, incorporating the J_2 gravitational perturbation represented by the additional term on the right-hand side of the equations, the equations of motion become

$$\ddot{x} = \frac{\partial V}{\partial x} = -\frac{\mu x}{r^3} + 3\frac{J_2 \mu R_\oplus^2}{2}\left(\frac{x}{r^5}\right)\left(1 - \frac{5z^2}{r^2}\right), \tag{5}$$

$$\ddot{y} = \frac{\partial V}{\partial y} = -\frac{\mu y}{r^3} + 3\frac{J_2 \mu R_\oplus^2}{2}\left(\frac{y}{r^5}\right)\left(1 - \frac{5z^2}{r^2}\right), \tag{6}$$

$$\ddot{z} = \frac{\partial V}{\partial z} = -\frac{\mu z}{r^3} + 3\frac{J_2 \mu R_\oplus^2}{2}\left(\frac{z}{r^5}\right)\left(3 - \frac{5z^2}{r^2}\right). \tag{7}$$

The coordinate system is fixed to the xy plane that is defined by the Earth's equatorial plane.[5]

Through the use of both heliocentric and geocentric coordinate system propagations, the orbital trajectory of an asteroid body may be tracked over time to find the close-approach locations of an asteroid body with the Earth.

ORBITAL ENCOUNTER GEOMETRIES

There are two distinct phases of the asteroid's trajectory that are considered in this analysis - the heliocentric and the geocentric trajectories. When the body undergoes an encounter with the Earth, there are a number ways that its orbit will be affected. Getting a good understanding of the geometry of planetary close-approaches and the effect that they have on the orbital elements of the bodies that undergo them should assist with the task of predicting the resulting orbital trajectory of the body after a planetary flyby.

Basic Assumptions

The dynamical system under consideration in the following analysis consists of the Sun, a planet orbiting the Sun in a circular orbit, and an asteroid, viewed as a particle, that is on an eccentric and inclined orbit around the Sun that crosses the orbit of the planet. Assume the planet has an orbital radius $R = 1$, the product $k\sqrt{M} = 1$, where k is the Gaussian constant and M is the mass of the Sun, and the asteroid has orbital parameters $(a, e, i, \omega, \Omega)$. In order to have the asteroid cross the orbital path of the planet, the asteroid must meet the following criteria: $a(1-e) < 1 < a(1+e)$.

The frame of reference established for this analysis is centered on the planet, the x-axis points radially opposite to the Sun, the y-axis is the direction of motion of the planet itself, and the z-axis is completes the right-handed system by pointing in the direction of the planet's angular momentum vector. The three most important orbital elements used in the analysis are the heliocentric orbital elements: semi-major axis a, eccentricity e, and inclination i, of the approaching body.[7]

Pre- and Post-Encounter Geometry

Let $\vec{U} = (U_x, U_y, U_z)$ and U be the relative velocity vector and magnitude between the planet and the asteroid,[7] defined as

$$U = \sqrt{3 - \left[\frac{1}{a} + 2\sqrt{a(1-e^2)}\cos i\right]} \tag{8}$$

$$U_x = U \sin\theta \sin\phi \tag{9}$$

$$U_y = U\cos\theta \tag{10}$$

$$U_z = U \sin\theta \cos\phi \tag{11}$$

where θ and ϕ define the angles that define the direction of U by

$$\phi = \tan^{-1}\left[\pm\sqrt{\frac{2a-1}{a^2(1-e^2)}} - 1\frac{1}{\sin i}\right] \tag{12}$$

$$\theta = \cos^{-1}\left[\frac{1-U^2-1/a}{2U}\right] \tag{13}$$

where θ may vary between 0 and π, and ϕ between -π/2 and π/2. In terms of a, e, and i, the components of $\vec{U}$ are given by

$$U_x = \left[2 - \frac{1}{a} - a(1-e^2)\right]^{1/2} \tag{14}$$

$$U_y = \sqrt{a(1-e^2)}\cos i - 1 \tag{15}$$

$$U_z = \sqrt{a(1-e^2)}\sin i \tag{16}$$

After the asteroid has an encounter with the target planet, the $\vec{U}$ vector is rotated by an angle γ in the direction ψ, where ψ is the angle measured counter-clockwise from the meridian containing the $\vec{U}$ vector. The deflection angle γ is related to the encounter parameter b by

$$\tan\frac{1}{2}\gamma = \frac{m}{bU^2} \tag{17}$$

where m is the mass of the planet, in units of the Sun's mass. The angle θ after the encounter, denoted by θ', is calculated from

$$\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\psi \tag{18}$$

and, defining $\xi = \phi - \phi'$, we have

$$\sin\xi = \sin\psi \sin\gamma / \sin\theta' \tag{19}$$

$$\cos\xi = (\cos\gamma \sin\theta - \sin\gamma \cos\theta \cos\psi)/\sin\theta' \tag{20}$$

$$\tan\xi = \sin\psi \sin\gamma / (\cos\gamma \sin\theta - \sin\gamma \cos\theta \cos\psi) \tag{21}$$

$$\tan\phi' = (\tan\phi - \tan\xi)/(1 + \tan\phi \tan\xi) \tag{22}$$

Evaluating for the post-encounter variables θ' and ϕ', the values of a', e', and i' can be obtained accordingly. Figure 1 pictorially represents the relationship between the pre-encounter (U, θ, ϕ) and post-encounter (U', θ', ϕ') variables that make up the geometry of a body's encounter.[7]

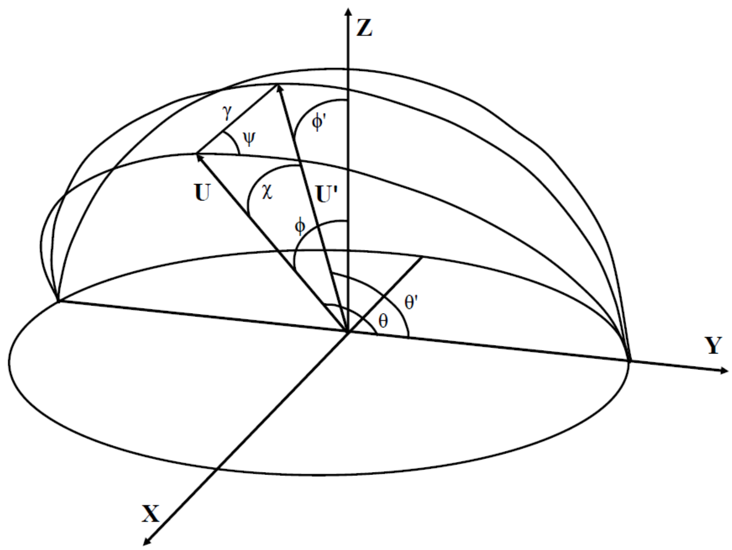

Figure 1. Reference frame of $\vec{U}$ and $\vec{U}'$. The origin is placed at the planet's center, the positive x-axis is opposite the direction of the Sun, the y-axis is in the direction of the planet's motion, and the z-axis is parallel to the planet's angular momentum vector. The angles ϕ and θ define the direction of $\vec{U}$. After an encounter with a planet, the vector $\vec{U}$ is rotated by and angle γ in the direction of ψ.

ANALYTIC KEYHOLE THEORY

Upon every encounter, the asteroid passes through what is known as a target plane. On these target planes can exist what are called keyholes, and if the asteroid were to pass through said keyhole, it would return on a resonant return orbit and impact the planet. The following describes the theory behind target planes and analytic keyhole computation.

Target Planes

A target plane is defined as a geocentric plane oriented to be normal to the asteroid's geocentric velocity vector. By observing the point of intersection of an asteroid trajectory with the target plane can lend significant insight into the nature of a future encounter. In general, there are two distinct planes and several coordinate systems that can be used in such a framework. The classical target plane is referred to as the B-plane, which has been used in astrodynamics since the 1960s. The

B-plane is oriented normal to the incoming asymptote of the geocentric hyperbola, or normal to the unperturbed relative velocity $\vec{v}_\infty$. The plane's name is a reference to the so-called impact parameter b, the distance from the geocenter to the intercept of the asymptote on this plane, known as the minimum encounter distance along the unperturbed trajectory.[9] Figure 2 depicts the relationship between the target B-plane and the trajectory plane of the asteroid.

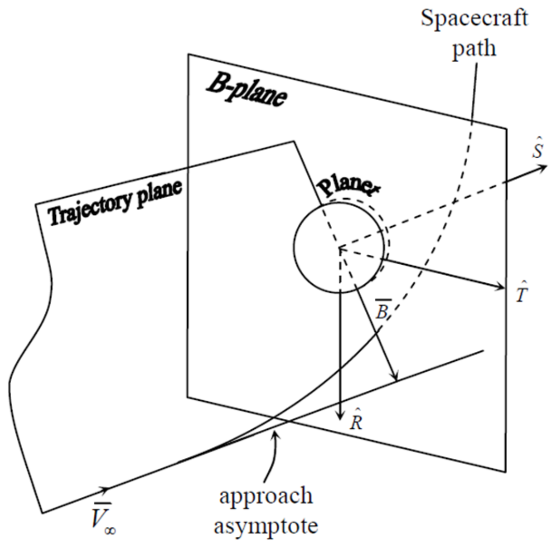

Figure 2. Representation of the target B-plane of a planet with respect to the incoming approach of a body on the trajectory plane. The coordinate axes depicted in the figure are just an example of those that could be used, not the ones used in the analyses.

Target Plane Coordinates

Generally it is convention to place the origin of the B-plane's coordinate system at the geocenter, but the orientation of the coordinate axes on the plane is arbitrary. The system has been fixed at times by aligning the axes in a way so that one of the nominal target plane coordinates is zero, or by aligning one of the coordinate axes with either the projection of the Earth's polar axis or the projection of the Earth's heliocentric velocity.

One of the most important functions of the target plane is to determine whether a collision is possible, and if not, how deep the encounter will be. With the B-plane, we obtain the minimum distance of the unperturbed asteroid orbit at its closest approach point with the Earth - the impact parameter b. That single variable however does not tell whether the asteroid's perturbed trajectory will intersect the image of the Earth on the following encounter, but the information can be extracted by scaling the Earth radius $R_\oplus$ according to the following relationship

$$b_\oplus = R_\oplus \sqrt{1 + \frac{v_e^2}{v_\infty^2}} \tag{23}$$

where v_e is the Earth escape velocity. With this formulation a given trajectory impacts the Earth if $b < b_\oplus$, and would not otherwise. Alternatively, the impact parameter could be scaled while leaving the image of the Earth on the B-plane unchanged. The two scalings are equivalent for a single orbit, but when computing the coordinates for different asteroids with different $\vec{v}_\infty$, the scaling is not uniform.[8]

A convenient and common target plane coordinate system (ξ, η, ζ) is obtained by aligning the negative ζ-axis with the projection of the Earth's heliocentric velocity $\vec{V}_\oplus$, the positive η-axis with the geocentric velocity (normal to the B-plane), and the positive ξ-axis in such a way that the reference frame is positively oriented, expressed as

$$\vec{\eta} = \frac{\vec{U}}{|\vec{U}|} \tag{24}$$

$$\vec{\xi} = \frac{\vec{\eta} \times \vec{V}_\oplus}{|\vec{\eta} \times \vec{V}_\oplus|} \tag{25}$$

$$\vec{\zeta} = \vec{\xi} \times \vec{\eta} \tag{26}$$

where $\vec{U}$ and U are the geocentric velocity vector and magnitude of the asteroid, respectively. With this reference frame, it can be seen that $\vec{\xi}$ and $\vec{\zeta}$ are on the B-plane itself, where (ξ, ζ) are the target plane coordinates that indicate the cross track and along track miss distances, respectively. That way, ζ is the distance in which the asteroid is early or late for the minimum possible encounter distance. The ξ coordinate, on the B-plane, refers to the minimum distance achieved by altering the timing of the encounter between the asteroid and the Earth, known as the Minimum Orbital Intersection Distance (MOID). It is important to note that this particular interpretation of the coordinates of the B-plane is only valid in the linear approximation, and unusable for distant encounters beyond several lunar distances.

Such a formulation of the problem gives rise to the thought that an asteroid can avoid impact if either the timing of the encounter is off or by being in an orbit that does not even intersect the Earth's orbit. Therefore, to have an impact occur the asteroid must have a small enough MOID and be on time for the encounter. The manner in which the encounters are characterized in this paper are according to the analytic theory developed by Valsecchi et al.[8]

Resonant Returns and Keyholes

A resonant return orbit is a consequence of an encounter with Earth, such that the asteroid is perturbed into an orbit of period $P' \approx k/h$ years, with h and k integers. After h revolutions of the asteroid and k revolutions of the Earth, both bodies are in the same region of the first encounter, causing a second encounter between the asteroid and the Earth.

The analytic theory of resonant returns that has been developed by Valsecchi et al.[8] treats close encounters with an extension of Opik's theory, adding a Keplerian heliocentric propagation between the encounters. The heliocentric propagation establishes a link between the outcome of the first encounter and the initial conditions of the next one. During the Earth encounter, the motion of the asteroid is assumed to take place on one of the asymptotes of the encounter hyperbola. The asymptote is directed along the unperturbed geocentric encounter velocity $\vec{v}_\infty$, crosses the B-plane at a right angle, and the vector from the Earth to the intersection point is denoted by $\vec{B}$.[9]

According to Opik's theory, the encounter of the asteroid with the Earth consists of the the instantaneous transition, when the body reaches the B-plane, from the pre-encounter velocity vector $\vec{v}_\infty$ to the post-encounter velocity vector $\vec{v}'_\infty$, such that $v'_\infty = v_\infty$. And, the angles θ' and ϕ' are simple functions of v_∞, θ, ϕ, ξ, and ζ, where θ is the angle between $\vec{v}_\infty$ and the Earth's heliocentric velocity $\vec{V}_\oplus$ and ϕ is the angle between the plane containing $\vec{v}_\infty$ and $\vec{V}_\oplus$ and the plane containing $\vec{V}_\oplus$ and the ecliptic pole. The deflection angle γ is the angle between $\vec{v}_\infty$ and $\vec{v}'_\infty$, described by

$$\tan\frac{\gamma}{2} = \frac{\kappa}{b} \tag{27}$$

where $\kappa = GM_\oplus/v_\infty^2$. In addition, simple expressions relate (a, e, i) to (v_∞, θ, ϕ), and (ω, Ω, ν) to (ξ, ζ, t_0), where t_0 is the time at which the asteroid passes the node closer to the encounter.[8,9]

A resonance orbit corresponds to certain values of a' and θ', that can be denoted by a'_0 and θ'_0. If the post-encounter is constrained in such a way that the ratio of periods between the Earth and the asteroid is k/h, then we have

$$a'_0 = \left(\frac{k}{h}\right)^{2/3} \tag{28}$$

$$\cos\theta'_0 = \frac{1 - U^2 - 1/a'_0}{2U} \tag{29}$$

$$= \cos\theta\,\frac{b^2 - c^2}{b^2 + c^2} + \sin\theta\,\frac{2c\zeta}{b^2 + c^2} \tag{30}$$

Thus, for a given U, θ, and θ'_0, we have

$$\cos\theta'_0 = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\psi \tag{31}$$

in the pre-keyhole B-plane, which gives the locus of points leading to a given resonant return.

If we solve for $\cos\psi$ and use $\zeta = b\cos\psi$ we get

$$\zeta = \frac{(b^2 + \kappa^2)\cos\theta'_0 - (b^2 - \kappa^2)\cos\theta}{2\kappa\sin\theta} \tag{32}$$

Replacing b^2 with $\xi^2 + \zeta^2$ and rearranging we obtain

$$\xi^2 + \zeta^2 - \frac{2\kappa\sin\theta}{\cos\theta'_0 - \cos\theta}\zeta + \frac{\kappa^2(\cos\theta'_0 + \cos\theta)}{\cos\theta'_0 - \cos\theta} = 0 \tag{33}$$

Equation (33) is that of a circle centered on the ξ-axis. If R is the radius of the circle and D is the value of the ξ-coordinate of its center, then Eq. (33) becomes

$$\xi^2 + \zeta^2 - 2D\zeta + D^2 = R^2 \tag{34}$$

Thus, the circle is centered at $(0, D)$ with

$$D = \frac{\kappa\sin\theta}{\cos\theta'_0 - \cos\theta} \tag{35}$$

and has a radius

$$R = \left|\frac{\kappa\sin\theta'_0}{\cos\theta'_0 - \cos\theta}\right|. \tag{36}$$

The circle intersects the ζ-axis at the values

$$\zeta = D \pm R = \frac{c(\sin\theta \pm \sin\theta'_0)}{\cos\theta'_0 - \cos\theta} \tag{37}$$

which represents the extreme values that b can take for a given a'. The circle intersects the ξ-axis at

$$\xi = \pm\kappa\sqrt{\frac{\cos\theta + \cos\theta_0'}{\cos\theta - \cos\theta_0'}}, \tag{38}$$

and the maximum value of $|\xi|$ for which a given θ_0' is accessible is R. The maximum value of a' accessible for a given U is for θ_0' = 0, and is obtained for

$$\zeta = \frac{\kappa\sin\theta}{1 - \cos\theta} \tag{39}$$

and the minimum value of a' is for θ_0' = π, and is obtained for

$$\zeta = -\frac{\kappa\sin\theta}{1 + \cos\theta} \tag{40}$$

In both cases we must have ξ = 0, meaning that this occurs for zero local MOID.

The intersection between the MOID and the resonance circles is a location for potential keyholes that could result future Earth impacts. Depending on the asteroid's arrival conditions, the asteroid could be put into one of those resonance orbits. It is important to note that if the resonance circle does not extend out far enough to intersect the MOID, then the potential of the asteroid entering into such a resonance orbit upon encounter can be neglected. The term 'keyhole' is used to indicate small regions of the B-plane of a specific close encounter so that if the asteroid passes through one of those regions, it will hit the Earth on the next return. An impact keyhole is one of the possible pre-images of the Earth's cross section on the B-plane tied to the specific value for the post-encounter semi-major axis that allows the subsequent encounter at the given date.[9]

To obtain the size and shape of an impact keyhole we can model the secular variation of the MOID as a linear term affecting ξ'' (the value of ξ at the next encounter)

$$\xi'' = \xi' + \frac{d\xi}{dt}(t_0'' - t_0'), \tag{41}$$

where t_0' and t_0'' are the times of passage at the node, on the post-first-encounter orbit that are closest to the first and second encounter, respectively. The time derivative of ξ can be calculated either by a secular theory for crossing orbits or by deduction from a numerical integration scheme. To compute the size and shape of the size and shape of the impact keyhole we start from the image of the Earth on the B-plane of the second encounter, and we denote the coordinates axes in this plane as ξ'', ζ''. The circle is centered on the origin and has a radius $b_\oplus$. The points on the target plane of the first encounter that are mapped to the points of the Earth image circle on the second encounter B-plane constitute the Earth pre-image that we are looking for.[8]

EXAMPLE - ASTEROID 2012 TC4

Many near-Earth asteroids exist that undergo close-encounters with the Earth. However, there are a significantly smaller number of near-Earth asteroids that have multiple, successive close-encounters with the Earth - asteroid 2012 TC4 is a good example of such an asteroid. The asteroid undergoes two successive Earth encounters within a five-year time period, the first of which occurred on October 12, 2012, and the second to occur on October 12, 2017. Asteroid 2012 TC4 is an Apollo class asteroid, meaning that the asteroid has a semi-major axis greater than that of the Earth's and perihelion distance smaller than the aphelion distance of the Earth. With an absolute magnitude (H) of 26.7 the diameter of 2012 TC4 would be about 12-27 meters, fairly small in comparison to most near-Earth asteroids.[10]

Table 1. Orbital elements of asteroid 2012 TC4 for its pre-first-encounter trajectory. Epoch - October 12, 2011.

Orbital Element	Value
a	1.2839 AU
e	0.29365
i	1.4074°
Ω	198.741°
ω	233.639°
ν_0	115.304°

Orbital Simulations

With the first of the two Earth encounters already having occurred, the position and the velocity of the asteroid body is very well known prior to October 2012. So, in order to capture the true dynamics of the encounters the asteroid simulations will begin one year prior to the October 12, 2011 encounter and would propagate through the encounter and proceed past the subsequent encounter with Earth on October 12, 2017, and concluding at the end of the October 2017 Earth flyby encounter. The analyses discussed in this paper were conducted using 100,001 virtual asteroid Monte Carlo simulation.

Given the known asteroid state prior to the October 2012 Earth encounter, shown in Table 1 the uncertainties in the states are prescribed at that point to be very small - 1 cm position uncertainty and 0.5 mm/s velocity uncertainty in all three coordinate directions. The state variations in the six asteroid states are shown in Figure 3. The state variations are shown with respect to the nominal

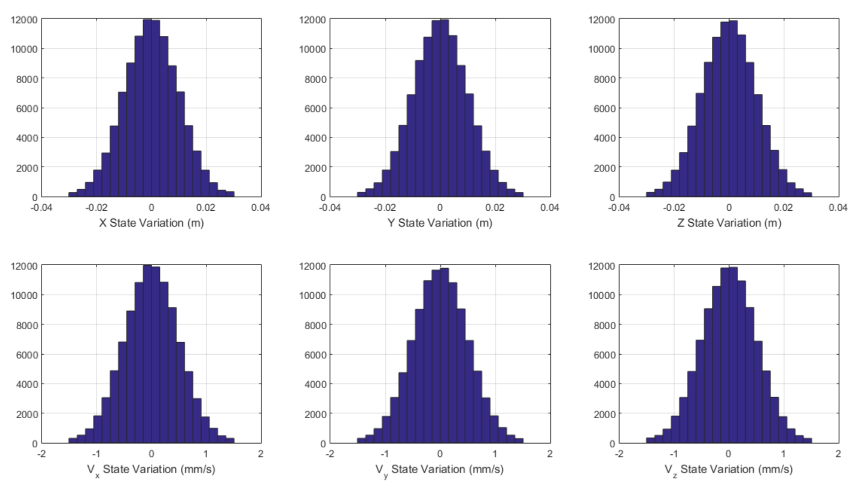

Figure 3. Histograms depicting the state vector variations in the 2012 TC4 asteroid cloud at the epoch of October 12, 2011 (the start of the simulation).

asteroid's state.

It is understood that the stated uncertainties are rather small for an asteroid, however given that the known state and trajectory prior to the first Earth encounter and the highly dynamic environment that the asteroid will experience, by passing near the Earth on two separate occasions, the small uncertainties are desired in order to have a consistent first encounter and ensure a high number of virtual asteroids will encounter the Earth on the second encounter in 2017.

Earth Encounter - October 2012

The first encounter with Earth that asteroid 2012 TC4 has occurred around October 12, 2012, one year after the initial epoch of the asteroid cloud. Of the 100,001 virtual asteroids in the Monte Carlo simulation, only 57 of the virtual asteroids do not experience an encounter with the Earth in October 2012. That number is interesting to note because despite the very small state variations, on the initial conditions set from a year prior to the encounter, not every virtual asteroid meets the Earth's sphere of influence on the first flyby. Beyond just the number of virtual asteroids that do not encounter Earth in 2012, the variation in the state vectors of those that do undergo an Earth flyby are non-Gaussian, as seen in Figure 4.

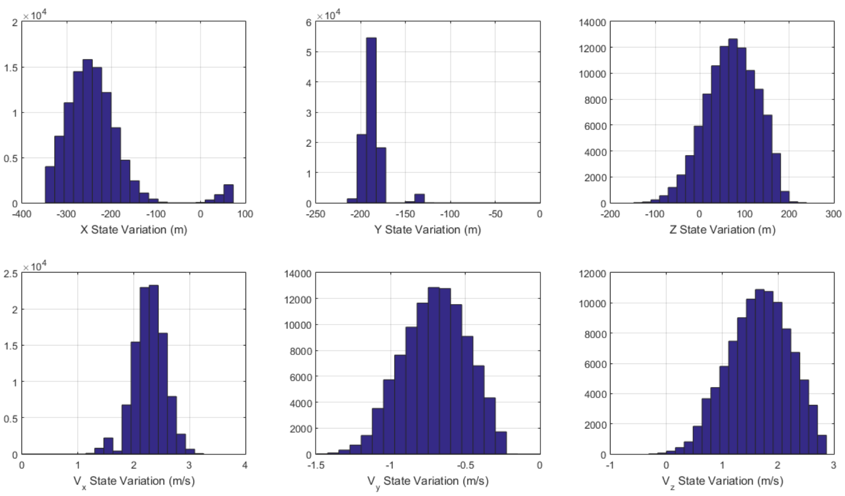

Figure 4. State variations of the virtual asteroid cloud with respect to the nominal asteroid state at close-approach in the October 2012 Earth encounter.

Looking at the different states, the Z-position and the three velocity components are the states that are the most Gaussian, but are noticeably skewed left (negative skew). The other two position components are clearly non-Gaussian, with the X-position showing a tendency towards a bimodal distribution. Looking at the value of the state variations, with respect to the nominal state vector, shows how much the virtual asteroid cloud has spread out over the first year of its propagation. The position variation of the asteroid cloud has expanded from tens of centimeters to hundreds of meters in all three coordinate directions. Similarly, the velocity variation of the cloud went from millimeter per second differences in speed to meters per second. So, entering the highly dynamic environment of Earth's sphere of influence, where small changes in position and/or velocity will become exaggerated after the Earth flyby, the state variations will only increase after the October 2012 encounter with the Earth.

After encountering the Earth in October 2012, asteroid 2012 TC4 falls into a resonance orbit that results in a second successive encounter with the Earth in October 2017. Before looking at the numerical propagation of the asteroid cloud between the two Earth encounters, analytic keyhole theory can be used to gain a better understanding of the asteroid cloud's encounter with the Earth. Figure 5 shows a depiction of the 2012 Earth encounter B-plane with the asteroid cloud and the asteroid's resonance circles.

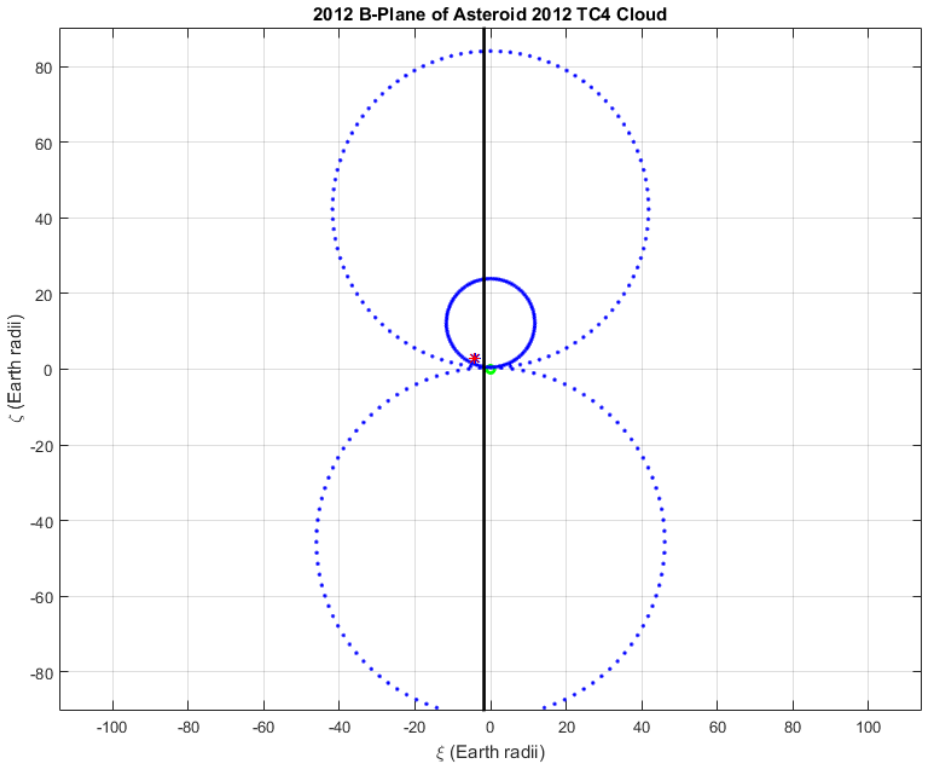

Figure 5. Depiction of the October 2012 Earth encounter B-plane, along with the asteroid 2012 TC4 cloud and resonance circles.

Based on analytic keyhole theory, five potential resonances exist for asteroid 2012 TC4 on the 2012 B-plane. In Figure 5, only three resonance circles are shown because the smallest resonance circle represents three different resonance circles that are equivalent - having the same size and location on the B-plane. Looking at the B-plane, it can be seen that the asteroid cloud location, with respect to the resonance circles and potential keyhole locations, is fairly close to those possible keyholes. If a closer look is taken at the B-plane plot, it can be seen that the cloud crossing locations are about one to two Earth radii from the possible keyhole locations. However, crossing through the analytically constructed keyhole on the encounter B-plane does not guarantee that the virtual asteroid would impact the planet, just like not passing through/near a keyhole region doesn't guarantee that a virtual asteroid won't re-encounter the planet. Continuing the orbital propagation of the virtual asteroid cloud will reveal not only the number of these virtual asteroids that encounter the Earth again in five years, but the depth of those encounters as well.

Earth Encounter - October 2017

To have a second, successive encounter with a planetary body, an asteroid must have an encounter with the body that results in a resonance return orbit. After the first encounter, the asteroid's resonance return orbit can have a different size and shape to than the original orbit that resulted in the original encounter. The orbital trajectories of asteroid 2012 TC4, both pre-first encounter and post-first/pre-second encounter, are shown with respect to Earth in Figure 6.

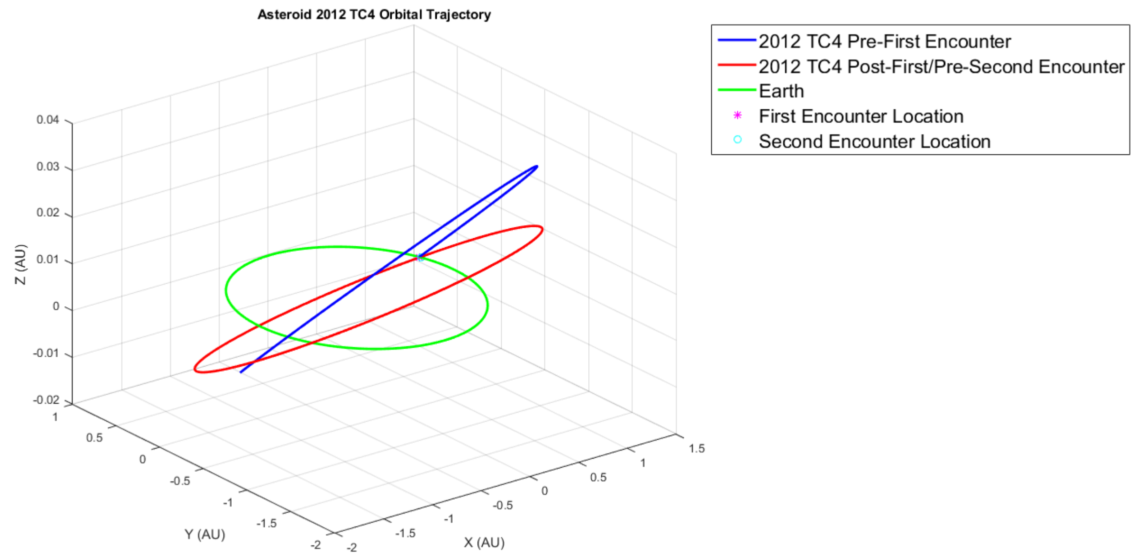

Figure 6. Depiction of the nominal orbital trajectory of asteroid 2012 TC4 with respect to the Earth. The Earth's trajectory is shown in green, the pre-first encounter trajectory of asteroid 2012 TC4 shown in blue, and the post-first/pre-second encounter trajectory of asteroid 2012 TC4 shown in red. The encounter locations for the first and second encounters are also identified by a magenta star and cyan circle, respectively, located at the intersection of the blue, red, and green orbits on the far-side of the grid.

The pre-first encounter, heliocentric asteroid orbit is shown in blue, and the orbital elements depicting its size and shape are semi-major axis of 1.284 AU, eccentricity of 0.294, and inclination of 1.407 degrees. After the encounter with Earth in October 2012, the orbit of asteroid 2012 TC4 gets altered in both size and shape. The semi-major axis and eccentricity of the asteroid increases to 1.406 AU and 0.294, respectively, but the inclination decreases to 0.856 degrees with respect to the ecliptic plane.

All 100,001 virtual asteroids that were propagated through their first encounter with the Earth in October 2012, were then propagated further forward through the October 2017 Earth encounter. Of the nearly 100,001 virtual asteroids that encountered the Earth in 2012, almost 991,000 of the virtual asteriods make a second encounter with the Earth in 2017. Unlike the seemingly similar Earth encounters by the virtual asteroid cloud on the October 2012 B-plane, the encounters by the cloud on the October 2017 B-plane are quite varied in terms of their location on the B-plane and their depth of encounter. Figure 7 shows the October 2017 B-plane and the encounter locations of the 2012 TC4 asteroid cloud. Looking at the encounter B-plane, the wide variation in the encounter locations of the virtual asteroid cloud can be seen. In the ζ direction, which is representative of the time of the asteroid's encounter, the location variations only span about 20 Earth radii, while the encounter locations in the ξ direction span nearly 200 Earth radii. Given the limited span of the encounters in the ζ direction after the five year propagation time since the first Earth encounter, with respect to the ξ direction, it would imply that there is not much of a variation in the velocity states of the virtual asteroid cloud. Looking at the state vector variations at Earth closest approach (Figure 8),

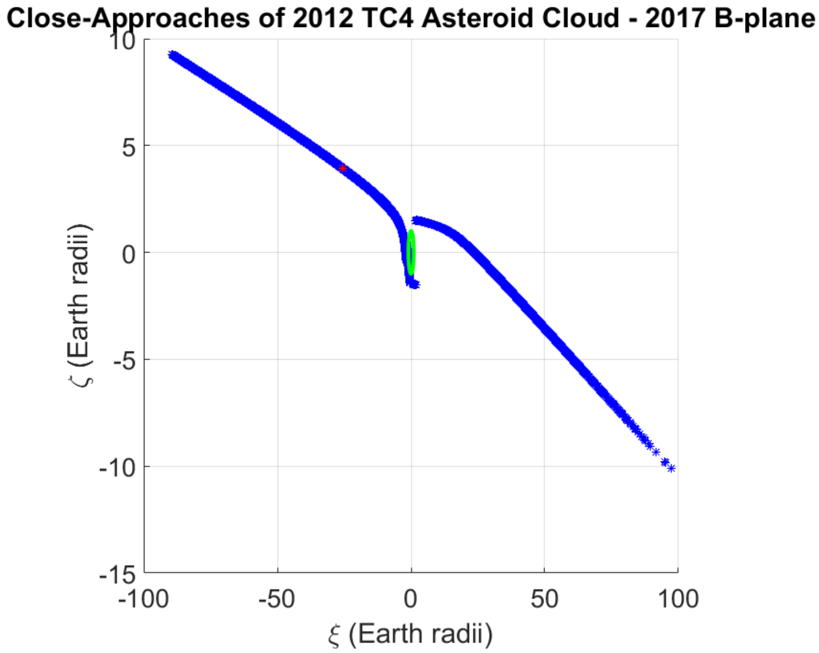

Figure 7. Depiction of the October 2017 Earth encounter B-plane, along with the asteroid 2012 TC4 cloud. The red star depicts the nominal asteroid's B-plane crossing location.

with respect to the nominal asteroid orbit, it can be seen that while there is some variation in the three velocity directions, a large majority of the virtual asteroids very similar velocity components. However, it is clear that despite the minimal variation on the first encounter B-plane, the Earth encounter has provided a wide positional state variation - particularly in the out-of-plane direction due to that change in inclination. That change in the heliocentric orbital inclination is responsible for the larger variation in the Z-component of the state vector. The Z-component has about five times the span of both the X- and Y-components.

Simply observing the relative state variations of the virtual asteroid cloud to the nominal asteroid state does not give a good depiction of the kinds of encounters the virtual asteroids will have with the Earth. Figure 9 shows the orbital element variations of the virtual asteroid cloud. The sixth plot of the virtual asteroid cloud's true anomaly angle shows that all the virtual asteroids are at a true anomaly of about zero degrees. The fourth and fifth plots show the variation in the longitude of the ascending node and the argument of periapse, respectively. Based on the histograms of those two variables, where most of the asteroids have very similar values, it appears that the asteroid orbit planes are very similarly oriented with respect to the Earth's equatorial plane. The three variables that define the shape and size of the orbits are different stories however. The histogram of the orbital inclination shows a majority of the virtual asteroids having an inclination between 100 and 110 degrees, but there are many virtual asteroids that have inclinations as small as 80 degrees and as large as about 165 degrees. Any body that is making an encounter with a planet will have a hyperbolic flyby of that planet, and the histograms of semi-major axis and eccentricity show exactly that. The eccentricity histogram shows a large amount of variation among the virtual asteroids, with a right-skewed distribution that has the right tail reaching out all the way to an eccentricity of 90. The semi-major axis histogram is the most normal of all the orbital element distributions, but

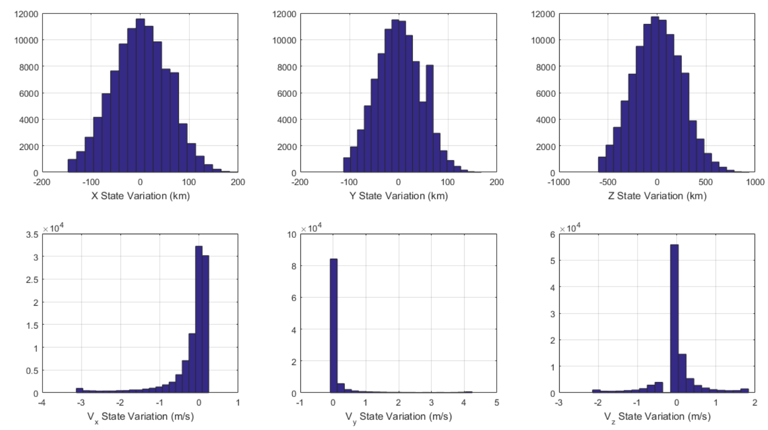

Figure 8. Orbital state vector variations of asteroid 2012 TC4 virtual asteroid cloud, with respect to the asteroid's nominal state, at Earth's closest-approach.

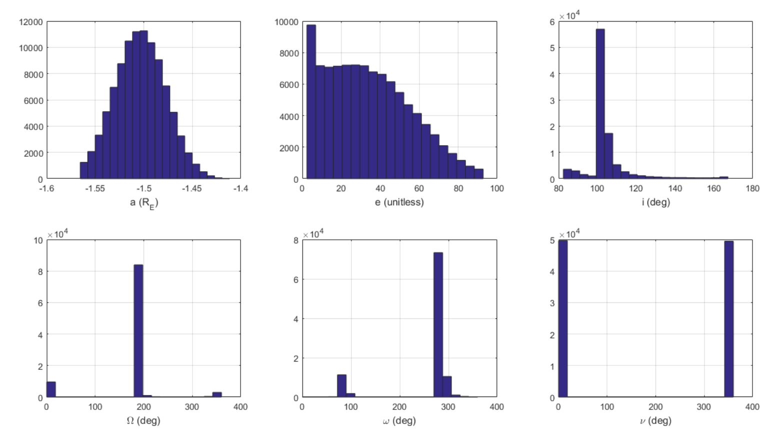

Figure 9. Orbital element variations of asteroid 2012 TC4 virtual asteroid cloud at Earth's closest-approach.

is skewed a little bit to the right. All the semi-major axis values are negative and the eccentricity values are all greater than one, meaning that none of the virtual asteroids are captured by the Earth and will continue in their heliocentric orbits.

IMPACT RISK ANALYSIS

Asteroid 2012 TC4 is a near-Earth asteroid that within a five-year time period has two close encounters with the Earth. The first encounter, in October 2012, is depicted by the B-plane plot in Figure 5. The virtual asteroid cloud has a deep encounter with the Earth, and passes near a potential keyhole location on the B-plane. After the 2012 encounter with Earth, asteroid 2012 TC4 completes four complete revolutions around the Sun before encountering the Earth again in October 2017. Due to the first encounter, the virtual asteroid cloud dispands and elongates along the orbit track leading to the diverse encounters on the October 2017 B-plane, shown in Figure 7.

The analytic keyhole theory briefly discussed earlier in this paper can tell us the size and location of the keyholes on a given B-plane, but they are rough estimates at best - given the assumptions that have to be made in order to calculate the resonance circles, B-plane crossing locations, and other values. By holding the orbit of the asteroid constant and varying the time of the encounters, a plot known as a keyhole map can numerically assess the size and location of potential keyholes on a given B-plane by propagating a field of virtual asteroids through at least two successive planetary encounters.

Utilizing the high-fidelity gravitational model described earlier in this paper, a field of virtual asteroids are propagated from a year prior to the October 2012 encounter through the October 2017 encounter. The keyhole map is constructed by plotting the ζ-component of the first encounter B-plane crossing locations versus the radial distance from the Earth on the later encounter B-planes. The reason that it is the later encounter B-planes is because the asteroid body can have multiple later encounters, and all those Earth radial distances are plotted simultaneously. On the keyhole map, each encounter will be viewed as a downward pointing spike on the plot, so each spike can indicate the location of the keyhole on the first encounter B-plane and the width of the spike when it crosses the one Earth radius radial distance line on the keyhole map tells the width of the keyhole on that first encounter B-plane. The keyhole map for asteroid 2012 TC4, for the October 2012 and October 2017 Earth encounters, is shown in Figure 10.

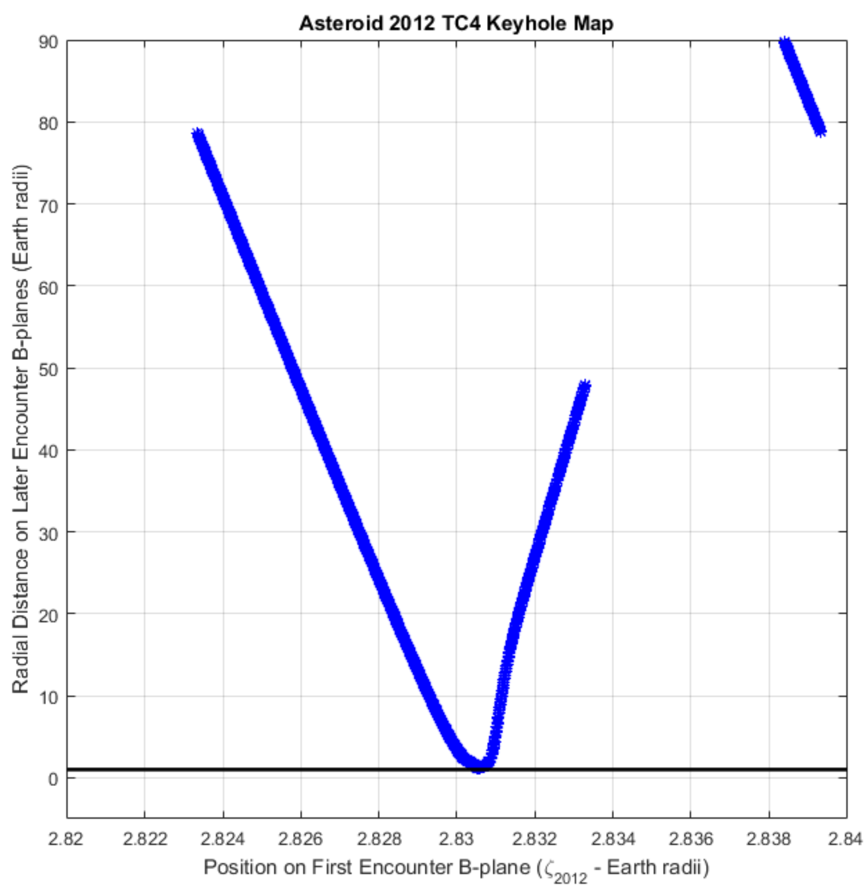

Figure 10. Depiction of the keyhole map of asteroid 2012 TC4. The blue stars indicate the positions and locations of the virtual asteroid field on the October 2012 and October 2017 B-planes. The black, horizontal line represents the Earth's surface on the keyhole map.

Looking closely at the keyhole map, it can be seen that the ζ-coordinate values span a pretty narrow range (less than 0.02 Earth radii). The narrow range is due to the time at which the virtual asteroid cloud is propagated from (one year prior to the October 2012 B-plane encounter). To ensure that the entire virtual asteroid field has a second encounter with the Earth in October 2017, the time variation of the field is fairly narrow. Similarly to the encounter analyses conducted in the previous section, the limited field variation, while it restricts the types of first encounters had by the field, there is a wide variation of encounters on the second encounter - a span of radial distances at close-approach as large as 90 Earth radii and as small as about 1.2 Earth radii. None of the virtual asteroids in the simulated field make a deep enough second encounter with the Earth to impact the planet. Therefore, without the spike in the keyhole map dipping below the one Earth radius line, it can be said that even with the calculated existance of a nearby keyhole on the October 2012 B-plane, asteroid 2012 TC4 will not pass through the keyhole and come back to impact the planet in October 2017.

CONCLUSIONS

Near-Earth asteroids and the risks associated with them is a problem that needs further study. Currently, there are two main approaches - analytic theory and numerical investigations. In this paper, the utilization of both analytic and numerical methods is used to assess the orbit and associated impact risk of asteroid 2012 TC4. The nominal orbit of asteroid 2012 TC4 has two close-encounters with Earth in a five-year timespan - the first encounter in October 2012 and the second encounter in October 2017. Propagating a cloud of 100,001 normally distributed virtual asteroids from a year prior to the first Earth encounter through the first encounter results in skewed set of states at close-approach in October 2012. Based on the 2012 B-plane plot, the virtual asteroid cloud crosses near a potential keyhole on the target B-plane. Based on the analytic theory, the cloud doesn't come close enough to the possible keyhole location, based on its estimated size and shape, for many, if any, of the virtual asteroids to pass through the keyhole and end up on an impacting trajectory with the Earth. However, the apparent lack of potential for a keyhole passage does not inhibit the asteroid cloud from falling into a resonance orbit and encountering the Earth again. The heliocentric orbit of asteroid 2012 TC4 after the first encounter with the Earth increases in both semi-major axis length and eccentricity and decreases in inclination. The resulting orbit is in a 4:5 resonance with Earth, where the asteroid completes four complete orbits around the Sun in the time that it takes the Earth to complete five orbits. Of the virtual asteroids that have a second encounter with the Earth, the close-approach, encounter states vary much more than those of the first encounter. The risk of the asteroid cloud impacting the Earth on the second encounter is fairly minimal, the nominal trajectory actually passes by the Earth harmlessly, but there are a fair number of virtual asteroids that pass relatively close to the Earth. Expanding upon the multiple, successive encounter analysis already conducted, a keyhole map is constructed based on the locations of the encounters on the first and second B-planes by varying the time parameter of the orbiting body. The keyhole map shows that while the analytic theory calculates the existance of a keyhole on the October 2012 B-plane, asteroid 2012 TC4 does not encounter/pass through the keyhole on the target B-plane. The results of the keyhole map imply that if the orbital states of the asteroid body are accurate, and only the time aspect of the orbit (meaning the location of the body in the orbit) is incorrect, then the asteroid won't pass through the keyhole on the October 2012 B-plane and won't impact the Earth when and if it encounters the Earth again in October 2017. More analyses would need to be done to ensure that there is no risk posed by the asteroid on Earth, but the initial analyses conducted in this paper show that the asteroid poses little risk to the planet in either of its encounters.

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