Terminal-Phase Guidance and Control Analysis of Asteroid Interceptors

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High-speed intercept missions, which include kinetic impactors and nuclear penetration devices, may be required to mitigate the threat from a near-Earth object. Various guidance laws, including the pulsed proportional navigation (PPN) guidance and advanced predictive guidance, are examined for the autonomous terminal-phase guidance and control of asteroid interceptors. The asteroid Apophis is used as an illustrative example to show that current technology can be used for high-speed intercept missions. A mission scenario based on a 2-dimensional orbital intercept model is simulated using classical, modified, and predictive guidance laws. Simulations show that intercept is possible even using the simple PPN guidance law for small target asteroids with an acceptable margin of error. The mission scenario is also simulated with a high-fidelity simulation program, called CLEON. The study results verify the applicability of the various guidance laws examined, and also confirm the suitability of the CLEON software for asteroid intercept mission design.

Nomenclature

- $a$ = semimajor axis of orbit
- $e$ = eccentricity
- $i$ = orbital inclination
- $n$ = mean orbital rate
- $r_a$ = aphelion
- $r_p$ = perihelion
- $t$ = current time
- $t_f$ = nominal end-of-mission time
- $t_{go}$ = nominal remaining mission time (time-to-go)
- $t_p$ = time of perigee passage
- $y$ = range to target
- $A_c$ = commanded acceleration for intercept spacecraft
- $A_T$ = target acceleration
- $\tilde{A}_S$ = gravitational acceleration acting on spacecraft
- $\tilde{A}_T$ = gravitational acceleration acting on target
- $M_0$ = mean anomaly at epoch
- $N$ = effective navigation ratio
- $V_a$ = aphelion velocity
- $V_c$ = closing velocity

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The potential threat to planet Earth from the impact of an asteroid has been recognized for a few decades. The scientific community has been engaged in identifying and characterizing hazardous near-Earth objects, and studying various techniques to mitigate the threat. One class of threat mitigation techniques is the high-speed interceptor. Intercept missions include the simple kinetic impactor, which alters the asteroid’s trajectory via momentum exchange, as well as more sophisticated penetrators to deliver, for example, nuclear explosives below the surface of the target asteroid. Effective interceptors must reliably hit a target asteroid at relative speeds of up to 10 km/s. NASA’s Deep Impact mission has demonstrated the possibility of an impact mission with a relative impact velocity of 10 km/s, albeit with a large target of 5 km in diameter.

Interceptor missions end with a terminal phase during which corrective maneuvers are executed to compensate for errors in initial orbital injection. The terminal phase begins some time after the on-board sensors can see the target and ends with intercept. For this study, the terminal phase is assumed to begin one day prior to intercept.

Navigation during the terminal phase of the mission will have to be autonomous, as computing and commanding accelerations via a communications link with Earth will be too time-consuming. The interceptor must be able to issue guidance commands based solely upon on-board measurements. The quality of on-board
measurements possible has been investigated in Refs. 2 and 3. Traditional GNC algorithms often require continuous and throttleable thrust levels, however most real divert thrusters are not throttleable, but rather have only two output values: on or off. Therefore, guidance schemes that command constant-thrust maneuvers are desirable. The classical proportional navigation (PN) guidance scheme and an augmented version to account for gravity are considered as a baseline GNC logic for an asteroid interceptor in this paper. A simple pulsed scheme based on proportional navigation is described, then a predictive guidance scheme based on orbital perturbation theory is investigated. Predictive guidance calculates impulsive velocity corrections at pre-determined times which ideally eliminates the final miss distance. Two predictive formulations are discussed, each based on different types of on-board measurements.

The asteroid Apophis is chosen as an illustrative example of a potentially hazardous near-Earth asteroid. Apophis has garnered much attention from the scientific community because it will make a number of close approaches to the Earth, with a slight chance of an Earth impact on April 13, 2036.

An intercept of Apophis one day after perihelion is simulated using the different guidance laws described. Simulations show an acceptable miss distance for all of the guidance laws for such a high-velocity mission. Intercept missions with a relative velocity of 3 km/s or more are considered to be hypervelocity missions. In the present work, an intercept velocity of 10 km/s is chosen as a worst-case scenario to test the limits of each guidance algorithm. The intercept scenario is also simulated using a high-fidelity simulation program, GMV’s CLEON software package. CLEON simulates intercept missions with realistic sensor and thruster characteristics. Simulations performed with CLEON verify that the guidance laws examined are applicable to high-speed intercept missions, and that CLEON can be effectively and reliably used for detailed mission design.

II. System Model and Mission Scenario

A. Dynamical Model

A target asteroid is modeled as a point mass in a heliocentric Keplerian orbit described by

\[
\ddot{R}_T + \frac{\mu_\odot}{R_T^3} \dot{R}_T = 0
\]  

(1)

where \( R_T \) is the position vector of the target asteroid from the sun, and \( \mu_\odot = 1.32715 \times 10^{20} \) \( m^3/s^2 \) is the solar gravitational parameter. The heliocentric orbital motion of the interceptor spacecraft is described by

\[
\ddot{R}_S + \frac{\mu_\odot}{R_S^3} \dot{R}_S - \frac{\mu_\odot}{R^3} \ddot{r} = \ddot{f}
\]  

(2)

where \( R_S \) is the position vector of the spacecraft from the sun, \( \mu_\odot \approx 15.35 \) \( m^3/s^2 \) is an estimated gravitational parameter for Apophis, and \( \ddot{f} \) is the sum of the perturbing force vectors (per unit mass) acting on the spacecraft. In this paper only forces from the commanded accelerations are considered for trajectory propagation dynamics.

The relative target-spacecraft separation is described by

\[
\vec{R} = \vec{R}_T - \vec{R}_S
\]  

(3)

where \( \vec{R} \) is the position vector of the target from the spacecraft.

From Figure 1 it can be seen that

\[
\lambda = \arctan \frac{R_y}{R_x}
\]  

(4)

where \( \lambda \) is the line-of-sight (LOS) angle, and \((R_x, R_y)\) are the components of the relative position vector along the inertial \((x, y)\) coordinates. Differentiating this with respect to time gives

\[
\dot{\lambda} = \frac{R_x V_y - R_y V_x}{R^2}
\]  

(5)
where \( \dot{\lambda} \) is the LOS rate, \( R = (R_x^2 + R_y^2)^{1/2} \), and the relative velocity components are found, by differentiating the relative position components, as follows:

\[
V_x = \dot{R}_x \\
V_y = \dot{R}_y
\]

Similarly, the velocity components for the spacecraft and target are found as follows:

\[
V_{Sx} = \dot{R}_{Sx} \\
V_{Sy} = \dot{R}_{Sy} \\
V_{Tx} = \dot{R}_{Tx} \\
V_{Ty} = \dot{R}_{Ty}
\]

B. Asteroid 99942 Apophis

The asteroid 99942 Apophis is used as an example target asteroid in this paper. Apophis has garnered much attention due to the possibility of a so-called keyhole passage on April 13, 2029. The probability of keyhole passage is now estimated to be much lower than initial estimates, but Apophis is still of interest due to its future approaches near the Earth.

The terminal phase of a mission starts when measurements of the target can be taken, sometime after the target is in visible sensor range. During the terminal phase, velocity corrections are computed to correct for errors in the initial orbit injection. For the Apophis scenario in this paper, the terminal phase begins 24 hours before nominal intercept time.

For this analysis, Apophis will be treated as a sphere with a diameter of 270 m, as given in JPL’s small-body database. A guidance law will be considered successful if the final miss distance is less than 30 m with respect to the center of brightness. This allows some margin for the irregular shape of Apophis and adverse lighting conditions.

Apophis is an Aten-class asteroid with an orbital semi-major axis less than 1 AU. Its orbital period about the sun is 323 days. Table 1 gives the six classical orbital elements of Apophis in JPL 140 (heliocentric ecliptic J2000 reference frame at epoch JD 2455000.5 (2009-June-18.0) TDB). Other orbital characteristics are estimated as: perihelion \( r_p = 0.74606 \) AU, aphelion \( r_a = 1.09881 \) AU, perihelion speed \( V_p = 37.6 \) km/s, aphelion speed \( V_a = 25.5 \) km/s, perihelion passage time \( t_p = JD 245894.9 \) (2009-Mar-04.41), mean orbital rate \( n = 2.2515 \times 10^{-7} \) rad/s, and mean orbital speed = 30.73 km/s.
Table 1. Orbital elements of asteroid Apophis at Epoch 2455400.5 (2010-July-23.0) TDB. Source: JPL’s small-body database.

<table>
<thead>
<tr>
<th>Orbital Elements</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis $a$, AU</td>
<td>0.9223</td>
</tr>
<tr>
<td>Eccentricity $e$</td>
<td>0.1911</td>
</tr>
<tr>
<td>Inclination $i$, deg</td>
<td>3.3317</td>
</tr>
<tr>
<td>Perihelion argument $\omega$, deg</td>
<td>126.4186</td>
</tr>
<tr>
<td>Right ascension longitude $\Omega$, deg</td>
<td>204.4320</td>
</tr>
<tr>
<td>Mean anomaly $M_0$, deg</td>
<td>202.4953</td>
</tr>
</tbody>
</table>

III. Guidance Algorithms

Several different guidance algorithms are considered and compared in this paper. Classical proportional navigation is analyzed, as well as modifications to it based on both thruster limitations and operations in the sun’s gravity field. A predictive guidance scheme is also discussed.

A. Proportional Navigation

Proportional navigation (PN) guidance is considered for both its simplicity and its historical interest. PN guidance is the most widely used law in practice. Simple improvements to PN guidance include implementing the sun’s gravitational effect and accommodating thruster limitations.

1. Classical Proportional Navigation

Classical proportional navigation guidance is considered in this section. The acceleration commands for this guidance scheme are given simply by

$$A_c = NV_c \dot{\lambda} \hspace{1cm} (12)$$

where $A_c$ is the commanded acceleration, $N$ is a constant called the effective navigation ratio, and $V_c$ is the spacecraft-target closing velocity. The distance between the spacecraft and the target is, as before,

$$R = (R_x^2 + R_y^2)^{\frac{3}{2}} \hspace{1cm} (13)$$

The closing velocity can be defined as the negative rate of change of the distance between the target and the interceptor. Differentiating the above equation yields the closing velocity as

$$V_c = -\dot{R} = -\frac{(R_x \dot{V}_x + R_y \dot{V}_y)}{R} \hspace{1cm} (14)$$

The LOS rate, closing velocity, and the effective navigation ratio suffice to command accelerations. The accelerations are applied perpendicular to the instantaneous LOS. The components are thus given as

$$A_x = -A_c \sin \lambda \hspace{1cm} (15)$$
$$A_y = A_c \cos \lambda \hspace{1cm} (16)$$

where $(A_x, A_y)$ are the components of the commanded acceleration of the interceptor in the inertial $(x, y)$ coordinate system.

2. Augmented Proportional Navigation

Proportional navigation guidance generates control commands based on the observed LOS angle and LOS rate. Considering knowledge of the acceleration due to the sun’s gravity can reduce the total acceleration
requirements for the spacecraft. Ref. 4 gives a formulation for augmented proportional navigation (APN) guidance.

From Figure 2, and using a small-angle approximation, the LOS angle can be given as

$$\lambda = \frac{y}{R}$$  \hspace{1cm} (17)

where $y$ is the separation between the spacecraft and the target perpendicular to the fixed reference frame.

For a fixed time of flight, we can define

$$t_{go} = t_f - t$$ \hspace{1cm} (18)

where $t_{go}$ is the mission time-to-go, $t_f$ is the nominal end-of-mission time and $t$ is the current time.

Because the separation between the target and the impactor must be zero at the end of the flight for a successful impact, the range equation can be linearized to yield

$$R = V_c(t_f - t) = V_c t_{go}$$ \hspace{1cm} (19)

Substituting the above expression in the LOS angle expression, Eq. (17), gives

$$\lambda = \frac{yt_{go}}{V_c t_{go}^2}$$ \hspace{1cm} (20)

This equation can be differentiated to give

$$\dot{\lambda} = \frac{y + yt_{go}}{V_c t_{go}^2}$$ \hspace{1cm} (21)

Comparing with standard PN guidance, we have the acceleration command as

$$A_c = NV_c \dot{\lambda} = \frac{N(y + yt_{go})}{t_{go}^2}$$ \hspace{1cm} (22)
The Zero-Effort-Miss (ZEM) distance is the separation between the target and the interceptor at the end of the flight, absent any additional accelerations. With no accelerations, the interceptor and target will continue on straight-line trajectories. The components of the ZEM can thus be given as

\[(ZEM)_x = R_x + V_xt_{go}\]  \hspace{1cm} (23)
\[(ZEM)_y = R_y + V_yt_{go}\]  \hspace{1cm} (24)

where \((ZEM)_x, (ZEM)_y\) are the ZEM components along the inertial \((x, y)\) coordinates. In an engagement scenario, the ZEM perpendicular to the LOS is of interest. Using trigonometry, we have

\[(ZEM)_\perp = -(ZEM)_x \sin \lambda + (ZEM)_y \cos \lambda\]  \hspace{1cm} (25)

where \((ZEM)_\perp\) is the ZEM distance perpendicular to the LOS. The term in parentheses in the new guidance law, Eq. (22), can be seen to be equivalent to the total ZEM distance, which is accurate for a non-accelerating target. A constant acceleration can be added to the ZEM term, appropriate for accelerations due to gravity. Adding an acceleration term to the ZEM gives

\[(ZEM) = y + \dot{y}t_{go} + 0.5A_Tt_{go}^2\]  \hspace{1cm} (26)

where \(A_T\) is the acceleration of the target relative to the spacecraft.

Substituting this ZEM expression into the guidance law, Eq. (22), gives

\[A_c = \frac{N(y + \dot{y}t_{go} + 0.5A_Tt_{go}^2)}{t_{go}^2} = NV_c\dot{\lambda} + \frac{NA_T}{2}\]  \hspace{1cm} (27)

The above guidance law requires knowledge or estimation of the target’s acceleration. For an asteroid in a Keplerian orbit, this acceleration is a known function of the target’s position. The acceleration on the interceptor can be similarly found. From the basic orbital force equation we have

\[\ddot{A}_{Tx} = -\frac{\mu_\odot R_{Tx}}{(R_{Tx}^2 + R_{Tx}^2_y)^{3/2}}\]  \hspace{1cm} (28)
\[\ddot{A}_{Ty} = -\frac{\mu_\odot R_{Ty}}{(R_{Tx}^2 + R_{Tx}^2_y)^{3/2}}\]  \hspace{1cm} (29)
\[\ddot{A}_{Sx} = -\frac{\mu_\odot R_{Sx}}{(R_{Sx}^2 + R_{Sx}^2_y)^{3/2}}\]  \hspace{1cm} (30)
\[\ddot{A}_{Sy} = -\frac{\mu_\odot R_{Sy}}{(R_{Sx}^2 + R_{Sx}^2_y)^{3/2}}\]  \hspace{1cm} (31)

where \((\ddot{A}_{Tx}, \ddot{A}_{Ty})\) and \((\ddot{A}_{Sx}, \ddot{A}_{Sy})\) are the gravitational acceleration components on the target and spacecraft, respectively, along the inertial \((x, y)\) coordinates. From trigonometry,

\[\ddot{A}_{T\perp} = -\ddot{A}_{Tx} \sin \lambda + \ddot{A}_{Ty} \cos \lambda\]  \hspace{1cm} (32)
\[\ddot{A}_{S\perp} = -\ddot{A}_{Sx} \sin \lambda + \ddot{A}_{Sy} \cos \lambda\]  \hspace{1cm} (33)

where \((\ddot{A}_{T\perp}, \ddot{A}_{S\perp})\) are the gravitational acceleration components perpendicular to the LOS. For scenarios where the only accelerations are caused by gravity, the target’s acceleration as seen by the spacecraft is

\[A_T = \ddot{A}_{T\perp} - \ddot{A}_{S\perp}\]  \hspace{1cm} (34)

Substituting Eq. (34) into the augmented PN guidance law, Eq. (27), gives the augmented proportional navigation guidance law as

\[A_c = NV_c\dot{\lambda} + \frac{N}{2}(\ddot{A}_{T\perp} - \ddot{A}_{S\perp})\]  \hspace{1cm} (35)
3. Pulsed Proportional Navigation

Some divert thrusters have minimal or no throttling ability. Classical PN guidance requires thrusters to provide varying thrust levels, therefore these thrusters cannot directly employ PN guidance. A fixed-thrust level thruster can still make use of the PN guidance commands by utilizing the so-called Schmitt trigger.\textsuperscript{5} Using this guidance scheme, acceleration commands are calculated by the PN guidance law as before. The trigger commands the divert thrusters to turn on once the commanded acceleration exceeds a certain magnitude, chosen by the designer, and off when the commanded acceleration reaches a designer-chosen cutoff. With traditional PN guidance the LOS rate must reach zero for a successful intercept. Therefore the second cutoff is typically selected as zero. The trigger control logic for pulsed proportional navigation (PPN) guidance is shown in Figure 3. The Schmitt trigger can also be used for augmented PN guidance, giving an augmented pulsed proportional navigation (APPN) guidance.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{schmitt_trigger.png}
\caption{Schmitt trigger logic.}
\end{figure}

B. Predictive Guidance

Augmented PN guidance is able to modestly reduce fuel consumption by making use of the accelerations from the sun’s gravity. However, this strategy still requires throttleable thrusters and is not applicable directly for spacecraft with fixed-thrust engines. Pulsed PN guidance allows the use of constant-thrust divert thrusters, but does not account for gravity and therefore uses some additional fuel. APPN guidance combines the advantages of pulsed and augmented guidance, but it can still be improved upon. A guidance method known as predictive guidance can be used to command engine pulses that will ideally eliminate the ZEM.

Predictive guidance commands thrust based on a corrected LOS which takes into account the target’s future position. It is based on a linearized perturbation theory to simplify on-board computations. Predictive guidance requires only knowledge of the target’s orbit and on-board measurements.

1. Perturbation Theory for Guidance

Orbital perturbations are small differences between the actual position and velocity of a spacecraft and a known (usually two-body) reference orbit. In this section the traditional notation for perturbations in terms of position and velocity deviations are adopted. The next section relates these quantities to the spacecraft and target states.

Consider a target traveling on a known reference path, which is a function of time. At any time $t$ the target has position $\vec{r}_0(t)$ and velocity $\vec{v}_0(t)$. The interceptor can be described by its position $\vec{r}(t)$ and velocity $\vec{v}(t)$. The position and velocity deviations are defined as

\begin{align}
\delta \vec{r} &= \vec{r}_0(t) - \vec{r}(t) \\
\delta \vec{v} &= \vec{v}_0(t) - \vec{v}(t)
\end{align}

(36) (37)

It is assumed that during the terminal guidance phase these deviations are small, so that linearization techniques are valid. Figure 4 shows the perturbation geometry.
Define

\[ \delta r(t) = \begin{pmatrix} \delta r_x \\ \delta r_y \end{pmatrix} \]  \hspace{1cm} (38)

\[ \delta v(t) = \begin{pmatrix} \delta v_x \\ \delta v_y \end{pmatrix} \]  \hspace{1cm} (39)

\[ r(t) = \begin{pmatrix} r_x \\ r_y \end{pmatrix} \]  \hspace{1cm} (40)

\[ v(t) = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \]  \hspace{1cm} (41)

where \((\delta r_x, \delta r_y), (\delta v_x, \delta v_y), (r_x, r_y), \) and \((v_x, v_y)\) are the components of the position deviation, velocity deviation, position vector, and velocity vector from the sun, respectively, along the inertial \((x, y)\) coordinates.

Given the position deviation at a given time, \(t\), the position deviation at the nominal end-of-mission time, \(t_f\), can be approximated as

\[ \delta r(t_f) = \frac{\partial r_0(t_f)}{\partial r_0(t)} \delta r(t) + \frac{\partial r_0(t_f)}{\partial v_0(t)} \delta v(t) \]  \hspace{1cm} (42)

The second term on the right hand side is usually small in comparison to the first term for asteroid intercept missions. Furthermore, the velocity deviation is difficult to measure, as will be discussed in a later section. Therefore, only the first term is kept.

Using the following definitions

\[ t_{go} = t_f - t \]  \hspace{1cm} (43)

\[ \vec{r}_0(t) = r_{0x}(t) \hat{i} + r_{0y}(t) \hat{j} \]  \hspace{1cm} (44)

\[ r_0(t) = (r_{0x}(t) + r_{0y}(t))^\frac{1}{2} \]  \hspace{1cm} (45)

we can obtain an expression for the partial derivatives in Cartesian coordinates. (Ref. 7)

\[
\frac{\partial r_0(t_f)}{\partial r_0(t)} = \left( 1 + \frac{3\mu_d t_o^2 r_{0y}(t)}{2 r_0^5(t)} - \frac{\mu_s t_o^2}{2 r_0^3(t)} \right) - \frac{3\mu_d t_o^2 r_{0x}(t) r_{0y}(t)}{2 r_0^5(t)} \left( 1 + \frac{3\mu_d t_o^2 r_{0x}(t) r_{0y}(t)}{2 r_0^5(t)} - \frac{\mu_s t_o^2}{2 r_0^3(t)} \right)
\]  \hspace{1cm} (46)
The LOS is the vector connecting the interceptor and the target. Based on the above definitions this is simply $\delta \vec{r}$. The LOS unit vector is thus $\delta \hat{r}$. With no course corrections, the target will be at the position $\delta \vec{r}(t_f)$ at the nominal end-of-mission time, relative to the interceptor. Note that $\delta \vec{r}(t_f)$ represents the miss vector at the end of the mission. A velocity correction applied in that direction, with the same magnitude as the current rate of change of target and interceptor position, causes the position deviation to be zero at the end of the mission. The miss vector is the separation the guidance system must take into account. Thus, by definition

$$\delta r_c = \delta \vec{r}(t_f)$$

(47)

$$\delta \hat{r}_c = \frac{\delta r_c}{\delta r_c}$$

(48)

where $\delta r_c$ is the “corrected” LOS, and $\delta \hat{r}_c$ is the unit vector in the “corrected” LOS direction. These two terms are called the corrected terms because they give the LOS corrected to take into account the sun’s gravitational force.

The time derivative of the position deviation is simply

$$\dot{\delta \vec{r}}(t) = \frac{d}{dt}\delta \vec{r}(t)$$

(49)

where $\dot{\delta \vec{r}}(t)$ is the rate of change of the position deviation and $\dot{\delta \vec{r}}(t) = ||\dot{\delta \vec{r}}(t)||$.

Define the required velocity change as the difference between the new velocity deviation after the maneuver is applied and the velocity deviation before the maneuver is applied as

$$\Delta \vec{v} = \delta \vec{v}^+ - \delta \vec{v}^-$$

(50)

where $\Delta \vec{v}$ is the required velocity change, $\delta \vec{v}^+$ is the velocity deviation immediately after an impulsive maneuver, and $\delta \vec{v}^-$ is the velocity deviation immediately before an impulsive maneuver. The desired new velocity deviation, as described above, is

$$\delta \vec{v}^+ = \delta \vec{r}(t)\delta \hat{r}_c$$

(51)

The velocity deviation before the maneuver is simply the current velocity deviation.

$$\delta \vec{v}^- = \delta \vec{v}(t)$$

(52)

The predictive guidance equation is thus

$$\Delta \vec{v} = \delta \vec{r}(t)\delta \hat{r}_c - \delta \vec{v}(t)$$

(53)

Note that $\delta \hat{r}_c$ is considered to be a constant unit vector as it is a function of only the reference orbit and the actual spacecraft orbit. Because the perturbation theory is linearized, the calculated value of $\delta \hat{r}_c$ may change at different points along the orbit.

2. Predictive Impulsive Guidance

To implement the above guidance law, some observations are first made. The position and velocity deviations are simply the relative position and velocity. The rate of change of the position deviation is the closing velocity between the interceptor spacecraft and the target asteroid. New definitions include
\[ \vec{R}(t) = \delta \vec{r}(t) \] (54)
\[ \vec{V}(t) = \delta \vec{v}(t) \] (55)
\[ V_c(t) = \delta \dot{r}(t) \] (56)
\[ \vec{\Lambda}(t) = \frac{\vec{R}(t)}{R(t)} \] (57)
\[ \vec{\Lambda}_c(t) = \frac{\vec{R}(tf)}{R(tf)} \] (58)

where \( \vec{\Lambda} \) is the LOS unit vector and \( \vec{\Lambda}_c \) is the “corrected” LOS unit vector. For convenience, define

\[ G(t) = \frac{\mu \odot}{R_T^3(t)} \begin{pmatrix} 3R_T^2(t) - R_T^2(t) & 3R_T R_X(t) R_T Y(t) \\ 3R_T X(t) R_T Y(t) & 3R_T^2(t) - R_T^2(t) \end{pmatrix} \] (59)

Where \( G \) is the gravity gradient matrix along the reference orbit. By defining

\[ \vec{R}(t) = \begin{pmatrix} R_x(t) \\ R_y(t) \end{pmatrix} \] (60)

we can show that the relative position at the nominal end-of-mission time is

\[ \vec{R}(tf) = \frac{t_{go}^2}{2} G(t) \vec{R}(t) + \vec{R}(t) \] (61)

The velocity correction to implement is now

\[ \Delta \vec{V}_S = V_c \vec{\Lambda}_c - \vec{V} \] (62)

This guidance law is known as predictive impulsive (PI) guidance, because it makes full use of the relative state vectors to predict the final miss distance. The \( G \) matrix is a function of the target asteroid’s position at a given time. The target asteroid’s orbit should be known well enough to permit accurate computation of the gravity gradient matrix at any time. The relative position vector of the asteroid from the spacecraft is needed to compute the final miss vector. The relative velocity vector is required to compute the acceleration commands. The closing velocity can be found from the relative state information. Relative position and velocity vectors are usually estimated by using a Kalman filter.

The mission time-to-go comes from the nominal end-of-mission time. The predictive guidance law calculates the maneuver necessary to achieve intercept at that particular nominal time. The spacecraft can thus be slightly ahead or behind its nominal terminal-phase trajectory and still achieve intercept. The time-to-go can also be estimated from the closing velocity.

3. Kinematic Impulsive Guidance

If measurements or estimates of the relative states are not available, the optical LOS and LOS rate may be used with some further approximations. The closing velocity must be estimated. For a successful intercept, the distance to the target can be estimated as the closing velocity multiplied by the mission time-to-go. Define

\[ \vec{\Lambda}(t) = \begin{pmatrix} \Lambda_x(t) \\ \Lambda_y(t) \end{pmatrix} \] (63)
where \((\Lambda_x(t), \Lambda_y(t))\) are the components of the LOS vector along the inertial \((x, y)\) coordinates. The LOS unit vector can be obtained from the measured LOS angle as

\[
\Lambda(t) = \begin{pmatrix}
\cos \lambda(t) \\
\sin \lambda(t)
\end{pmatrix}
\] (64)

The relative position can then be estimated as

\[
R(t) = V_c t_{go} \Lambda(t)
\] (65)

Substituting Eq. (65) into Eq. (61) gives an estimated relative position at the nominal end-of-mission time as

\[
R(t_f) = \frac{V_c^3 t_{go}^2}{2} G(t) \Lambda(t) + V_c t_{go} \Lambda(t)
\] (66)

Define

\[
\dot{\Lambda}(t) = \begin{pmatrix}
\dot{\Lambda}_x(t) \\
\dot{\Lambda}_y(t)
\end{pmatrix}
\] (67)

\[
V(t) = \begin{pmatrix}
V_x(t) \\
V_y(t)
\end{pmatrix}
\] (68)

where \((\dot{\Lambda}_x(t), \dot{\Lambda}_y(t))\) and \((V_x(t), V_y(t))\) are the components of the rate of change of the LOS unit vector and the components of the relative velocity, respectively, along the \((x, y)\) inertial coordinates. Taking the time derivative of Eq. (64) gives

\[
\dot{\Lambda}(t) = \begin{pmatrix}
-\dot{\lambda}(t) \sin \lambda(t) \\
\dot{\lambda}(t) \cos \lambda(t)
\end{pmatrix}
\] (69)

The relative velocity can be approximated as a component along the LOS and a component in the LOS rate direction as

\[
V(t) = V_c t_{go} \dot{\Lambda}(t) + V_c \Lambda(t)
\] (70)

Recall the guidance law for predictive impulsive guidance, Eq. (62) described by

\[
\Delta \vec{V}_S = V_c \vec{\Lambda}_c - \vec{V}
\] (71)

Then, substituting Eq. (70) into Eq. (71) gives the kinematic impulsive (KI) guidance law as

\[
\Delta \vec{V}_S = V_c (\vec{\Lambda}_c - t_{go} \dot{\vec{\Lambda}} - \vec{\Lambda})
\] (72)

This guidance law is known as kinematic impulsive (KI) guidance because it makes use of the kinematics of the system, which must be estimated. The measured LOS angle and LOS rate must still be filtered, but simpler filters, such as a least-squares filter, can be used instead of a Kalman filter. The gravity gradient matrix is again a known function of time, but the final miss vector calculation now depends on the closing velocity, and is more dependent on the time-to-go. The guidance law itself again requires the closing velocity and time-to-go. In addition to optical measurements, good estimates of the closing velocity and time-to-go are essential. The kinematic guidance law also allows for missions slightly ahead or behind the nominal time, but the guidance law computation itself is more affected by inaccuracies in estimating time-to-go.
IV. Simulation Results

A. 2-D Simulation

The guidance laws described in Section III were simulated with a 4th-order Runge-Kutta numerical integration scheme. The asteroid Apophis is at perihelion at the beginning of the terminal phase. The interceptor spacecraft is ahead of the target and displaced outward radially. This scenario puts extreme demands on the GNC algorithms, starting with a 10.8 km/s closing velocity. In the absence of guidance commands, the initial conditions will result in a miss distance of 40,000 km. The initial conditions are given in Table 2.

The baseline spacecraft is a 1000-kg interceptor spacecraft with 10-N thrusters. Thrusters are turned off for the final ten minutes of each mission scenario. For the basic simulations it is assumed that all measurements of positions in the heliocentric frame, relative positions, angles and angular rates, and the LOS are available with no errors.

Predictive guidance schemes use a pre-defined firing schedule for trajectory correction maneuvers. Earlier firing times require less Δv because the impactor trajectory has not deviated as far from the reference trajectory. However, measured or estimated relative states for realistic sensing equipment are less reliable when the spacecraft is further from the target. Later firing times benefit from improved measurements, but require larger corrective maneuvers to overcome position deviations. Because the guidance theory itself is linearized, the calculated corrective maneuvers at any given time will, in general, not be accurate. The first maneuver in particular is not sufficient by itself for intercept, as the approximations from linearization are increasingly detrimental with increasing position and velocity deviations. Therefore multiple firings are required to assure successful intercept. The linearized results are more accurate on some parts of the trajectory than on others. However, there is no way to know when better firing times are, as the accuracy of the linearization changes for different orbits. Finally, the performance of the kinematic impulsive scheme degrades near the end of the mission, as the further linearization for relative position causes increased error when the interceptor approaches the target.

To again test the guidance schemes in a worst-case scenario, only three firing times are selected. The first firing takes place with 20 hours to go, the second with 9 hours to go, and the third with 10 minutes to go. These times encompass an early firing for a large burn to put the spacecraft much closer to an intercept course, a burn approximately halfway through the terminal phase when the linearization errors are much smaller, and a final burn shortly before impact. The firing sequence was chosen based on the three-pulse adaptive scheme used by the Deep Impact mission.9

Table 3 gives the Δv requirements and final miss distance for each guidance law. Trajectories of the spacecraft and target, LOS and LOS rate histories, and Δv usage are shown in Figures 5-14.

The proportional navigation and augmented PN guidance schemes behave as expected described in Ref. 4. Continuous thrust is commanded during the entire mission in both cases, with the augmented system requiring slightly less Δv. Given the short flight time compared to the target’s orbital period, and the distance of the spacecraft and asteroid from the sun, augmented PN acceleration commands do not differ significantly from PN commands.

The pulsed PN and APN guidance schemes calculate the acceleration required for standard PN and APN, respectively, but only apply acceleration when commanded by the Schmitt trigger logic. Pulsed guidance is able to achieve impact, but at the cost of possibly increased Δv compared to that required for PN/APN guidance. Pulsed PN/APN guidance is seen to require less Δv than standard PN/APN guidance does for the scenario studied. Pulsed guidance requires more Δv for non-accelerating targets, but a scenario with a constantly accelerating target could require either more or less, depending on the initial conditions.

The predictive guidance schemes are able to achieve intercept with only three thruster firings. They do not require the large number of short pulses near the end of the flight that are characteristic of pulsed guidance schemes.

<table>
<thead>
<tr>
<th>body</th>
<th>x position, km</th>
<th>y position, km</th>
<th>x velocity, km/s</th>
<th>y velocity, km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>111.626×10^6</td>
<td>0</td>
<td>0</td>
<td>37.6301</td>
</tr>
<tr>
<td>Interceptor</td>
<td>111.627×10^6</td>
<td>936.358×10^3</td>
<td>0</td>
<td>26.7926</td>
</tr>
</tbody>
</table>
Table 3. Performance comparison of various guidance laws

<table>
<thead>
<tr>
<th>Guidance Laws</th>
<th>$\Delta v$, m/s</th>
<th>ZEM, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN</td>
<td>5.6704</td>
<td>4.2725</td>
</tr>
<tr>
<td>Augmented PN</td>
<td>5.5470</td>
<td>3.4257</td>
</tr>
<tr>
<td>Pulsed PN</td>
<td>4.7214</td>
<td>4.3745</td>
</tr>
<tr>
<td>Pulsed Augmented PN</td>
<td>4.2342</td>
<td>3.3390</td>
</tr>
<tr>
<td>Predictive Impulsive</td>
<td>4.3351</td>
<td>3.3876</td>
</tr>
<tr>
<td>Kinematic Impulsive</td>
<td>4.3394</td>
<td>4.8713</td>
</tr>
</tbody>
</table>

Figure 5. Trajectories, Line-of-Sight Angle, Commanded Acceleration, and Applied Acceleration for PN guidance.
Figure 6. Closing Velocity, Line-of-Sight Rate, $\Delta v$ Used, and Position Difference for PN guidance.

Figure 7. Trajectories, Line-of-Sight Angle, Commanded Acceleration, and Applied Acceleration for APN guidance.
Figure 8. Closing Velocity, Line-of-Sight Rate, $\Delta v$ Used, and Position Difference for APN guidance.

Figure 9. Trajectories, Line-of-Sight Angle, Commanded Acceleration, and Applied Acceleration for PPN guidance.
Figure 10. Closing Velocity, Line-of-Sight Rate, $\Delta v$ Used, and Position Difference for PPN guidance.

Figure 11. Trajectories, Line-of-Sight Angle, Commanded Acceleration, and Applied Acceleration for APPN guidance.
Figure 12. Closing Velocity, Line-of-Sight Rate, $Δv$ Used, and Position Difference for APPN guidance.

Figure 13. Trajectories, Line-of-Sight Angle, Line-of-Sight Rate, and Detailed View of Rate for PI guidance.
Figure 14. Closing Velocity, Commanded Acceleration, $\Delta v$ Used, and Position Difference for PI guidance.

Figure 15. Trajectories, Line-of-Sight Angle, Line-of-Sight Rate, and Detailed View of Rate for KI guidance.
B. CLEON Software Simulation

1. Overview

GMV has developed a software (SW) tool called CLEON for high-fidelity simulation of the closed-loop trajectory of the asteroid interceptor.\(^3\) CLEON is a hybrid (continuous–discrete) multi-rate SW simulator implemented in Matlab/Simulink\(^\text{©}\). A block diagram of the program is shown in Figure 17. Two different levels of realism for the optical sensors (navigation camera and star-tracker) are available, and global-performance models are implemented for fast Monte Carlo analysis. Models are stored in a library allowing for fast changes in the simulator, for instance addition of redundant sensors or substituting an existing component by a new one.

From the Graphical User Interface (GUI), the user can edit all the configuration files and perform some simple mission analysis to help in the selection of scenario parameters. In addition, the GUI allows the user to select different spacecraft (SC) configurations.

The GUI allows the selection of the type of simulation mode: Monte Carlo batch simulation with autotocoding capability, or single-run that opens the simulator model. Finally, the GUI allows easy management of the simulations database and visualization of the results of any stored simulation.

About the dynamics model, it is worth saying that the initial asteroid state is taken from JPL405 ephemerides and the trajectories of asteroid and SC are integrated independently in heliocentric coordinates. The accelerations acting on the SC, apart from Sun’s gravitation and divert thrust, include attitude control system thrust, solar radiation pressure and asteroid gravity.

CLEON includes the following models within the optical sensors for global-performance simulation of attitude (star-tracker) and LOS (navigation camera):

- The target magnitude results from the ideal magnitude with an added variation modeled as a Exponentially Correlated Random Variable (ECRV).
- The center of brightness (CoB) motion around the center of mass (CoM) can be (1) harmonic, (2) an ECRV, or (3) harmonic plus an ECRV.
• The LOS and attitude performances are computed including the following errors:
  – Mounting bias and (thermal) drift
  – Noise Equivalent Angle (NEA) that is function of:
    * Real image: exposure time, visual magnitude of target and stars in the field of view (FOV), sky magnitude,
    * Optics: transmittance, point spread function (PSF), stray light,
    * Detector: assumed to be a CCD (pixel size, quantum efficiency, fill ratio), and
    * Electronics: read-out noise, dark current
  – Centroid error and other miscellaneous sources of Gaussian error
• The navigation camera includes an extended-target error model.
• The image processing performs a stacking of the navigation camera observations in order to increase the signal-to-noise ratio of the final measurement.

![Simulation block diagram](image)

**Figure 17. Simulation block diagram.**

2. Image Processing

The image processing (IP) algorithm extracts the LOS information from the images affected by real camera effects. Initially, the near-Earth object is so faint that the detected photoelectrons from the target are small compared with the image bias or noise. In order to increase the signal-to-noise ratio of the electron count coming from the target, a stacking of images is carried out prior to computing the LOS from the SC towards the target CoB. The sequence of operations of the IP algorithm is the following:

• Camera calibration to remove the bias from raw images subtracting a master dark frame. The master dark frame is computed averaging a given number of raw images taken with the same exposure time and temperature of the detector and electronics than the navigation images.

• Stacking of calibrated images to make the target detectable against a grainy background. The calibrated images are co-aligned prior to stacking. The effect of the image stacking is equivalent to a longer exposition time, but the stacking is preferred because relaxes the constraints on the ACS and prevents saturation and blooming.

• Centroiding of the pixel counts in a search-box to find the direction to the CoB. Prior to computing the centroiding, the pixels fainter than a given threshold are filtered out and, optionally, the isolated bright spots are removed.
3. GNC Algorithms

Several navigation algorithms are implemented to estimate different parameters, but they all process the same inputs coming from the optical sensors. The available navigation algorithms are the following:

- For LOS navigation, i.e. to provide a smoothed LOS and LOS-rate, a digital fading memory filter proposed by Ref. 4 or a batch-sequential least-squares filter.
- For state navigation, i.e. to estimate the impactor state relative to the target, a sequential Kalman–Schmidt filter (or extended Kalman filter) formulated by Ref. 10.

The suite of autonomous guidance schemes that compute the divert maneuvers include:

- Predictive guidance, which computes an impulsive maneuver at a given time that ideally cancels the Zero-Effort-Miss (ZEM) with respect to the center of brightness (CoB). The gravity gradient of the sun is considered in the $\Delta v$ computation. The configuration parameters are the times for execution of the maneuvers.
- Proportional navigation, which computes an acceleration vector proportional to the LOS-rate and the homing velocity $V_c$, as presented in Eq. (12). The only configuration parameter is the navigation ratio.
- Hybrid scheme, which implements mid-course predictive guidance and terminal proportional-navigation. The configuration parameters are the execution times of the impulsive maneuvers, the start time of the proportional navigation and the navigation ratio.

The control algorithm transforms the computed inertial acceleration into burning time of each thruster, considering the number and orientation of the thrusters. In predictive guidance (impulsive maneuvers), the thrusters take some cycles to complete the commanded $\Delta v$ and the control computes the average acceleration in the next cycle using the accelerometer information. The saturation is not considered in the control algorithm but in the reaction control system model.

4. CLEON Simulation Results

Monte Carlo simulations using PN guidance and PI guidance were performed for the Apophis intercept scenario. The CoB-CoM offset is half of Apophis’s radius. For PI guidance, the firing schedule in Ref. 3 is adopted. Figures 18 and 19 show the cumulative distribution functions for ZEM and $\Delta v$, locations of the impact points in the B-plane, and the evolution of the ZEM. The ZEM distribution shows that all of the Monte Carlo shots were within 20 meters of the CoB for PN guidance, and within 15 meters for PI guidance. The $\Delta v$ distribution shows that all of the PN Monte Carlo shots required less than 9 m/s, with 90% requiring less than 6 m/s. For PI guidance, all of the Monte Carlo shots required less than 8 m/s, with 90% requiring less than 5 m/s. The plot of the impact points show all of the impacts clustering near the CoB. For an Apophis-sized asteroid with a CoB-CoM offset of half the body’s radius, the performance is acceptable for impacting the body. The impact radius evolution gives a measure of the effectiveness of the guidance law over time. PN guidance is seen to gradually and continuously decrease the expected impact radius, while PI guidance shows increasing accuracy with each maneuver.

V. Conclusion

The simulation results in this paper show that high-speed asteroid interceptor missions are possible with current GNC technology. Such high-speed intercept missions include kinetic impactors and nuclear penetrators. A mission scenario described in this paper considered a 1000-kg interceptor spacecraft with 10-N thrusters, the reference asteroid Apophis, and the sun. The scenario setup included an Apophis perihelion intercept with an intercept relative velocity of approximately 10 km/s. Proportional Navigation, Augmented PN, Predictive Impulsive, and Kinetic Impulsive guidance laws were examined individually in the scenario. Modifications and improvements can be made to the traditional PN guidance law such as adding a Schmitt trigger or incorporating the effects of gravity. Predictive guidance can improve the overall effectiveness of the guidance system while relying solely on on-board optical sensor instrumentation and information. All guidance laws were simulated in the proposed scenario and led to a spacecraft-asteroid intercept within an
Figure 18. ZEM cumulative distribution function, $\Delta v$ cumulative distribution function, impact points on target, and impact radius evolution for PN guidance.

Figure 19. ZEM cumulative distribution function, $\Delta v$ cumulative distribution function, impact points on target, and impact radius evolution for PI guidance
acceptable margin of error for the final miss distance. Simulations with the CLEON software package verify that the guidance laws can be employed with realistic thruster and sensor characteristics.

Pulsed PN guidance is able to achieve similar accuracy to predictive guidance, for a similar fuel cost. Pulsed PN guidance did not require more than three burns in this challenging scenario. Pulsed PN guidance only requires on-board optical measurements. Thus pulsed PN guidance can match the performance of the more complex predictive guidance schemes given the same measurement information. The simple pulsed guidance method can be used for asteroid intercept missions.

Simulation studies using a simple 2-dimensional intercept engagement model have also validated the suitability of CLEON for the terminal phase in real mission design work in the future. Moreover, a three-dimensional model with an attitude control system will allow for a more realistic simulation of the thrusters and the quality of sensor measurements. Simulated sensor information with errors will better demonstrate the effectiveness of the guidance schemes in a realistic application.

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References